Exercises, Day 1

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Names: ____

1 Geodesics on a sphere

Consider the surface of a sphere embedded in three-dimensional space.

(a) The straightest possible lines on this surface are the great circles. Show that among the circles all of whose points lie on the surface of the sphere, these are indeed the ones with the greatest radius (and thus the least curved, or straightest, as viewed as curves embedded in three-dimensional space).

(b) What is the relation between the circumference of a circle whose points all lie on the surface on the sphere and the circle's "radius along the sphere"? (Define "distance along the sphere" of two points on the spherical surface as the length of the great-circle arc connecting them. Define the circle's "center on the sphere" as that [or more concretely, one of two] point[s] on the sphere that has the same distance from all of the circle points.)

(c) Consider two great circles that start out parallel (that is, two of whose tangents are parallel — that is, are both at right angles to the same [third] great circle intersecting the two others). For a special example, show that the arcs of the two initial great circles converge to a point as you follow them along.

2 Geodesics in special relativity

(a) Using the Euler-Lagrange equations, show that time passing on the co-moving clock ("proper time") of a particle flying from an event (t_0, x_0) to another event (t_1, x_1) is extremal ("a geodesic (line)") whenever the particle is moving along a straight line in spacetime.

(b) The Euler-Lagrange equations can tell us only about stationarity, not about whether we are dealing with a minimum or a maximum. To see which is which, look at two possibilities for a (non-massless) particle moving. The first is the straight worldline linking events A and B, the second a worldline linking A and B which consists of two straight segments joined at point C. Which proper time interval is larger — that for the particle trajectory directly from A to B, or for the detour to C? Hint: As in many exercises involving relativity, you can make your life much easier by choosing a particularly suitable reference frame from the very beginning.)